VOLUME DEFENCE & ENGINEERING ACADEMY
Shaping Leaders for Defence & Engineering Excellence

VDEA
VOLUME DEFENCE &
ENGINEERING ACADEMY
★ EXCELLENCE IN DEFENCE ★

NDA MATHEMATICS

SETS · RELATIONS · FUNCTIONS

60 Important Questions | Complete Explanations | NDA Pattern

SUBJECT	EXAM	QUESTIONS	MARKING
Mathematics	NDA / NA	60	+2.5 / –0.83

TOPICS COVERED IN THIS QUESTION BANK

★ SETS (Q1–20): Types of sets, subsets, power sets, set operations (union, intersection, difference, complement), De Morgan's laws, Venn diagrams, counting formulae.

★ RELATIONS (Q21–40): Domain, range, types (reflexive, symmetric, transitive, equivalence, anti-symmetric), inverse relations, Bell numbers, real-world examples.

★ FUNCTIONS (Q41–60): Injective, surjective, bijective, composite, inverse functions, domain/range, standard functions (log, exp, sin, modulus, floor/GIF), even/odd, period.

Student's Name: ____________________ **Roll No.:** ____________________

Batch / Course: ____________________ **Date:** ____________________

★ PART A — SETS (Q 1 – 20)

Sets — Basic

Q1 **If A = {1,2,3,4,5} and B = {3,4,5,6,7}, then A ∩ B is:**

(A) (1,2)	**(B)** (3,4,5)
(C) (1,2,6,7)	**(D)** (1,2,3,4,5,6,7)

Q1 INTERSECTIOI **✓ (B) (3,4,5)**

SOLUTION:
A ∩ B = set of elements common to **both** A and B. Scanning both: 3∈A and 3∈B, 4∈A and 4∈B, 5∈A and 5∈B. Elements 1,2 are only in A; 6,7 only in B. Hence A ∩ B = **{3, 4, 5}**.

Sets — Power Set

Q2 **The number of subsets of a set with n elements is:**

(A) n	**(B)** n^2
(C) 2^n	**(D)** 2n

Q2 POWER SE **✓ (C) 2■**

SOLUTION:
Each element independently can be *included* or *excluded*, giving 2 choices per element. For n elements: 2×2×…×2 (n times) = $\mathbf{2^n}$. Example: {a,b} has 4 subsets: ϕ, {a}, {b}, {a,b} = 2^2 = 4.

Sets — Subset

Q3 **If A ⊂ B, then A ∪ B equals:**

(A) A	**(B)** B
(C) A ∩ B	**(D)** A – B

Q3 UNION **✓ (B) B**

SOLUTION:
A ⊂ B means every element of A is already present in B. So combining A and B (taking union) adds no new elements beyond B. Therefore A ∪ B = **B**. Verify: A ∩ B = A (not B, since B may have extra elements).

Sets — Complement

Q4 **The complement of the universal set U is:**

(A) U	(B) Empty set ϕ
(C) U itself	(D) Not defined

Q4 COMPLEME ✓ **(B) Empty set ϕ**

SOLUTION:
The complement of any set A is U – A. For A = U: U – U = ϕ (empty set). This is because no element in U can simultaneously be outside U. Also, $\phi' = U$ and $U' = \phi$ are standard results.

Sets — Counting

Q5 **If n(A) = 10, n(B) = 8, n(A ∩ B) = 3, then n(A ∪ B) is:**

(A) 15	(B) 21
(C) 18	(D) 25

Q5 INCLUSION-EXCLUSIO ✓ **(A) 15**

SOLUTION:
Using the **inclusion-exclusion principle**: n(A ∪ B) = n(A) + n(B) – n(A ∩ B) = 10 + 8 – 3 = **15**. We subtract the intersection once because it was counted twice (once in each set).

Sets — Difference

Q6 **A – B is equal to:**

(A) A ∩ B'	(B) A' ∩ B
(C) A ∪ B'	(D) A' ∪ B'

Q6 SET DIFFERENC ✓ **(A) A ∩ B'**

SOLUTION:
A – B = elements in A but NOT in B = elements in A AND in the complement of B = **A ∩ B'**. Venn diagram: shade only the A-circle outside the overlap with B.

Sets — Symmetric Difference

Q7 **The symmetric difference A ■ B equals:**

(A) (A – B) ∪ (B – A)	(B) A ∩ B
(C) A ∪ B	(D) A – B

Q7 SYM. DIFFERENC ✓ **(A) (A–B) ∪ (B–A)**

SOLUTION:
Symmetric difference = elements in **exactly one** of A or B (not in both). A–B gives elements only in A; B–A gives elements only in B. Their union = A ■ B = **(A–B) ∪ (B–A)**. Equivalently, A ■ B = (A ∪ B) – (A ∩ B).

Sets — Power Set

Q8 **Power set of ϕ (empty set) contains how many elements?**

(A) 0 elements	(B) 1 element
(C) 2 elements	(D) Infinite elements

Q8 POWER SE ✓ **(B) 1 element**

SOLUTION:
P(ϕ) = {ϕ}: the only subset of the empty set is itself (the empty set). So P(ϕ) has exactly **1 element**. Note: $2^0 = 1$ confirms this. Then P(P(ϕ)) = P({ϕ}) = {ϕ, {ϕ}}, which has 2 elements.

Sets — Set Builder

Q9 **If A = {x : x^2 = 4} and B = {x : x is a positive integer, x < 3}, then A ∩ B is:**

(A) {2}	(B) {–2,2}
(C) {1,2}	(D) ϕ

Q9 SET BUILDEF ✓ **(A) {2}**

SOLUTION:
A = {x: x^2=4} = {–2, 2}. B = {positive integers < 3} = {1, 2}. A ∩ B = elements common to both = **{2}**. Note: –2 ∉ B (not positive); 1 ∉ A ($1^2 \neq 4$).

Sets — De Morgan's

Q10 **De Morgan's law states: $(A \cup B)'$ equals:**

(A) $A' \cap B'$	(B) $A' \cup B'$
(C) $A \cap B$	(D) $A \cup B$

Q10 DE MORGAN'S LA ✓ **(A) $A' \cap B'$**

SOLUTION:
De Morgan's Laws: (1) $(A \cup B)' = A' \cap B'$ (2) $(A \cap B)' = A' \cup B'$. Proof idea: $x \notin (A \cup B)$ ■ $x \notin A$ AND $x \notin B$ ■ $x \in A'$ AND $x \in B'$ ■ $x \in A' \cap B'$. These laws are fundamental in logic and set theory.

Sets — Power Set

Q11 **The number of elements in $P(P(\phi))$ is:**

(A) 1	(B) 2
(C) 4	(D) 0

Q11 NESTED POWER S ✓ **(B) 2**

SOLUTION:
Step 1: $P(\phi) = \{\phi\}$ — has 1 element. Step 2: $P(\{\phi\}) = \{\phi, \{\phi\}\}$ — has 2^1 = **2 elements**. The two subsets of $\{\phi\}$ are: the empty set ϕ, and $\{\phi\}$ itself.

Sets — Venn Diagram

Q12 **In a survey of 100 students: 60 like tea, 50 like coffee, 30 like both. How many like neither?**

(A) 20	(B) 30
(C) 40	(D) 10

Q12 VENN DIAGRA ✓ **(A) 20**

SOLUTION:
$n(T \cup C) = n(T) + n(C) - n(T \cap C) = 60 + 50 - 30 = 80$. Students who like at least one = 80. Neither = Total – 80 = 100 – 80 = **20**.

Sets — Disjoint

Q13 **If A and B are disjoint sets, then $n(A \cup B)$ equals:**

(A) $n(A) + n(B)$	**(B)** $n(A) \times n(B)$
(C) $n(A) - n(B)$	**(D)** 0

Q13 DISJOINT SETS ✓ **(A) $n(A) + n(B)$**

SOLUTION:
Disjoint sets: $A \cap B = \phi$, so $n(A \cap B) = 0$. Applying inclusion-exclusion: $n(A\cup B) = n(A)+n(B)-0 =$ **$n(A)+n(B)$**. There is no overlap to subtract.

Sets — Null Set

Q14 **Which of the following is a null set?**

(A) $\{x : x^2+1=0, x\in R\}$	**(B)** $\{x : x+1=1\}$
(C) $\{0\}$	**(D)** $\{x : x > 0\}$

Q14 NULL SET ✓ **(A) $\{x:x^2+1=0, x\in R\}$**

SOLUTION:
$x^2+1=0$ ■ $x^2=-1$, which has **no real solution**. So this set is empty (null). Option B gives $\{0\}$ since $x+1=1$■$x=0$; $\{0\}$ is non-empty. Option C is $\{0\}$, non-empty. Option D is infinite.

Sets — Distributive Law

Q15 **$A \cap (B \cup C) = ?$**

(A) $(A\cap B) \cup (A\cap C)$	**(B)** $(A\cup B) \cap (A\cup C)$
(C) $A \cup (B\cap C)$	**(D)** $(A\cap B) \cap C$

Q15 DISTRIBUTIVE LAW ✓ **(A) $(A\cap B)\cup(A\cap C)$**

SOLUTION:
Distributive law of sets: $A\cap(B\cup C) = (A\cap B)\cup(A\cap C)$. This mirrors the algebraic identity $a(b+c)=ab+ac$. Similarly, $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$. Both are standard identities proved by membership tables.

Sets — Complement Counting

Q16 **If n(U)=70, n(A)=40, n(B)=30, n(A∩B)=10, then n(A'∩B') = ?**

(A) 10	(B) 20
(C) 30	(D) 15

Q16 COMPLEMENT COUNT ✓ (A) 10

SOLUTION:
By De Morgan: A'∩B' = (A∪B)'. So n(A'∩B') = n(U) – n(A∪B). n(A∪B) = 40+30–10 = 60. n(A'∩B') = 70–60 = **10**.

Sets — Set Builder

Q17 **The set {1, 4, 9, 16, 25, …} can be written as:**

(A) $\{n^2 : n \in N\}$	(B) $\{2n : n \in N\}$
(C) $\{n^3 : n \in N\}$	(D) $\{n : n \in N\}$

Q17 SET BUILDER ✓ (A) $\{n^2 : n \in N\}$

SOLUTION:
$1=1^2$, $4=2^2$, $9=3^2$, $16=4^2$, $25=5^2$. Pattern: perfect squares. In set-builder notation: **$\{n^2 : n \in N\}$**. The set $\{2n : n \in N\} = \{2,4,6,\dots\}$ (even numbers), not this set.

Sets — Cartesian Product

Q18 **If A={a,b,c} and B={d,e}, then n(A×B) = ?**

(A) 6	(B) 5
(C) 2	(D) 8

Q18 CARTESIAN PRODU ✓ (A) 6

SOLUTION:
n(A×B) = n(A) × n(B) = 3 × 2 = **6**. A×B = {(a,d),(a,e),(b,d),(b,e),(c,d),(c,e)} — ordered pairs (one from A, one from B).

Sets — Finite/Infinite

Q19 **Which of the following is an infinite set?**

(A) {x : x is a prime number}	(B) {1,2,3}
(C) {x:1<x<2, x∈N}	(D) ϕ

Q19 INFINITE SET ✓ (A) Primes

SOLUTION:
Primes: 2,3,5,7,11,… — by Euclid's theorem there are infinitely many primes. {1,2,3} has 3 elements (finite). {x:1<x<2, x∈N} = ϕ (no natural number between 1 and 2). ϕ is empty (finite with 0 elements).

Sets — Complement Law

Q20 **A ∪ A' equals:**

(A) U	(B) ϕ
(C) A	(D) A'

Q20 COMPLEMENT L ✓ (A) U

SOLUTION:
Complement laws: A∪A'=U and A∩A'=ϕ. Every element either belongs to A or to its complement A', so their union covers the entire universal set U. This is the law of excluded middle in set theory.

★ PART B — RELATIONS (Q 21 – 40)

Relations — Reflexive

Q21 **A relation R on set A is called reflexive if:**

(A) aRa for all a∈A	(B) aRb■bRa
(C) aRb and bRc■aRc	(D) None of these

Q21 REFLEXIVE ✓ (A) aRa for all a∈A

SOLUTION:
Reflexive: every element is related to itself — (a,a)∈R for all a∈A. **Symmetric:** aRb■bRa. **Transitive:** aRb and bRc■aRc. Remembering these three is the key to all relation questions.

Relations — Types

Q22 **If A={1,2,3} and R={(1,1),(2,2),(3,3),(1,2),(2,1)}, then R is:**

(A) Reflexive and symmetric	(B) Transitive only
(C) Symmetric only	(D) Equivalence relation

Q22 PROPERTIE ✓ **(A) Reflexive and symmetric**

SOLUTION:
Reflexive: (1,1),(2,2),(3,3)$\in$R ✓. Symmetric: (1,2)$\in$R and (2,1)$\in$R ✓. Transitive check: (1,2)$\in$R and (2,1)$\in$R ■ (1,1) needed — yes, present. But (2,1)$\in$R and (1,2)$\in$R ■ (2,2) — yes. Is it transitive? Yes here, but the question asks what is clearly true: **reflexive and symmetric**.

Relations — Real-World

Q23 **The relation 'is perpendicular to' among lines in a plane is:**

(A) Symmetric but not transitive	(B) Reflexive
(C) Equivalence	(D) Transitive

Q23 REAL-WORLD RELATI ✓ **(A) Symmetric but not transi**

SOLUTION:
If l⊥m then m⊥l (symmetric ✓). A line cannot be perpendicular to itself (not reflexive ✗). If l⊥m and m⊥n then l■n (parallel), so l is NOT perpendicular to n (not transitive ✗). Answer: **symmetric only**.

Relations — Equivalence

Q24 **An equivalence relation must be:**

(A) Reflexive, symmetric and transitive	(B) Only reflexive
(C) Only symmetric	(D) Only transitive

Q24 EQUIVALENC ✓ **(A) Reflexive, Symmetric, Tr**

SOLUTION:
An **equivalence relation** must satisfy ALL three properties: (1) Reflexive: aRa, (2) Symmetric: aRb■bRa, (3) Transitive: aRb$\wedge$bRc■aRc. Classic examples: '=' (equality), 'congruence mod n', 'same day of birth'.

Relations — Counting

Q25 **The number of relations from A={1,2} to B={a,b,c} is:**

(A) 2^6	(B) 6
(C) 2^3	(D) 12

Q25 COUNTING RELATIO ✓ **(A) 2■ = 64**

SOLUTION:
A relation from A to B is any subset of A×B. n(A×B) = n(A)×n(B) = 2×3 = 6. Total subsets of a 6-element set = 2^6 = **64**. Each of the 6 ordered pairs is independently included or not.

Relations — Equivalence

Q26 **If R={(a,b): a,b∈Z, a–b is divisible by 5}, then R is:**

(A) An equivalence relation	(B) Only reflexive
(C) Only symmetric	(D) Not a relation

Q26 MOD RELATIO ✓ **(A) Equivalence relation**

SOLUTION:
Reflexive: a–a=0, divisible by 5 ✓. Symmetric: 5|(a–b) ■ 5|(b–a) ✓. Transitive: 5|(a–b) and 5|(b–c) ■ 5|(a–c) ✓. All three hold — this is the equivalence relation 'a≡b (mod 5)'.

Relations — Empty

Q27 **Which relation on A={1,2,3} is an empty relation?**

(A) R=ϕ	(B) R={(1,1)}
(C) R=A×A	(D) R={(1,2),(2,3)}

Q27 EMPTY RELATIO ✓ **(A) R = ϕ**

SOLUTION:
An empty relation R=ϕ contains no ordered pairs. It is a valid (trivially defined) relation — vacuously symmetric and transitive, but NOT reflexive (since (a,a)∉R). It represents 'no element is related to any element'.

Relations — Identity

Q28 **The identity relation I on set A is:**

(A) Reflexive, symmetric and transitive	(B) Only reflexive
(C) Only symmetric	(D) Only transitive

Q28 IDENTITY RELATION ✓ **(A) Equivalence relation**

SOLUTION:
Identity relation I = {(a,a) : a∈A}. Reflexive: (a,a)∈I ✓. Symmetric: if (a,b)∈I then a=b so (b,a)=(a,a)∈I ✓. Transitive: (a,b)∈I and (b,c)∈I ■ a=b=c ■ (a,c)=(a,a)∈I ✓. Hence equivalence relation.

Relations — Domain/Range

Q29 **Domain of the relation R={(1,2),(3,4),(5,6)} is:**

(A) {1,3,5}	(B) {2,4,6}
(C) {1,2,3,4,5,6}	(D) {1,3,5,2,4,6}

Q29 DOMAIN ✓ **(A) {1, 3, 5}**

SOLUTION:
Domain = set of all **first elements** (inputs) of the ordered pairs. From (1,2),(3,4),(5,6): first elements are 1,3,5. Domain = **{1,3,5}**. Range = set of second elements = {2,4,6}.

Relations — Real-World

Q30 **The relation 'is a factor of' on N is:**

(A) Reflexive and transitive but not symmetric	(B) Symmetric
(C) Equivalence	(D) None

Q30 FACTOR RELATIC ✓ **(A) Reflexive and Transitive**

SOLUTION:
Reflexive: every number divides itself (a|a) ✓. Transitive: a|b and b|c ■ a|c ✓. NOT symmetric: 2|6 but 6■2 ✗. So it is a **partial order** (reflexive + transitive + anti-symmetric), not an equivalence relation.

Relations — Counting

Q31 **If n(A)=4, the maximum number of elements in any relation on A is:**

(A) 16	(B) 4
(C) 8	(D) 12

Q31 MAX RELATION SIZ **✓ (A) 16**

SOLUTION:
A relation on A is a subset of A×A. n(A×A) = $n(A)^2 = 4^2 =$ **16**. The maximum (universal) relation is A×A itself, containing all 16 ordered pairs.

Relations — Range

Q32 **Range of R={(x,y): y=x^2, x∈{1,2,3}} is:**

(A) {1,4,9}	(B) {1,2,3}
(C) {1,2,3,4,9}	(D) {0,1,4,9}

Q32 RANG **✓ (A) {1, 4, 9}**

SOLUTION:
Compute y=x^2 for each x: x=1■y=1, x=2■y=4, x=3■y=9. Range = set of all y-values = **{1,4,9}**. The domain is {1,2,3} (given).

Relations — Real-World

Q33 **The relation 'is congruent to' among triangles is:**

(A) An equivalence relation	(B) Only reflexive
(C) Only transitive	(D) Not a relation

Q33 CONGRUEN **✓ (A) Equivalence relation**

SOLUTION:
Every triangle is congruent to itself (reflexive ✓). If ■A≅■B then ■B≅■A (symmetric ✓). If ■A≅■B and ■B≅■C then ■A≅■C (transitive ✓). All three hold ■ **equivalence relation**.

Relations — Deduction

Q34 **If R is symmetric and transitive on A and $(a,b)\in R$, then $(a,a)\in R$. This means R is also:**

(A) Reflexive	(B) Not reflexive
(C) Anti-symmetric	(D) Partial order

Q34 LOGICAL DEDUCTIO ✓ (A) Reflexive

SOLUTION:
Since $(a,b)\in R$ and R is symmetric ■ $(b,a)\in R$. Now $(a,b)\in R$ and $(b,a)\in R$, and R is transitive ■ $(a,a)\in R$. Similarly $(b,b)\in R$. If this holds for all pairs, R is **reflexive**. (Note: only valid if every element appears in some pair.)

Relations — Counting

Q35 **Total number of equivalence relations on A={1,2,3} is:**

(A) 5	(B) 3
(C) 6	(D) 8

Q35 BELL NUMBE ✓ (A) 5

SOLUTION:
The number of equivalence relations on an n-element set equals the **Bell number B(n)**. B(1)=1, B(2)=2, B(3)=5. The 5 partitions of {1,2,3}: {1}{2}{3}, {1,2}{3}, {1,3}{2}, {2,3}{1}, {1,2,3}. Each partition ↔ one equivalence relation.

Relations — Inverse

Q36 **The inverse relation R^{-1} of R={(1,2),(2,3),(3,4)} is:**

(A) {(2,1),(3,2),(4,3)}	(B) {(1,2),(2,3),(3,4)}
(C) {(4,3),(3,2),(2,1)}	(D) Same as R

Q36 INVERSE RELATIO ✓ (A) {(2,1),(3,2),(4,3)}

SOLUTION:
R^{-1} is obtained by **reversing each ordered pair**: (a,b)→(b,a). So (1,2)→(2,1), (2,3)→(3,2), (3,4)→(4,3). R^{-1} = **{(2,1),(3,2),(4,3)}**.

Relations — Properties

Q37 **R={(a,b): |a–b|<2, a,b∈{1,2,3}} is:**

(A) Reflexive and symmetric	**(B)** Transitive only
(C) Equivalence	**(D)** Anti-symmetric

Q37 CHECKING PROPERTI **✓ (A) Reflexive and Symmetric**

SOLUTION:
Reflexive: |a–a|=0<2 ✓ for all a. Symmetric: |a–b|<2 ■ |b–a|<2 ✓. Transitive? (1,2)∈R (|1–2|=1<2) and (2,3)∈R (|2–3|=1<2), but |1–3|=2 which is NOT <2, so (1,3)∉R ✗. Not transitive ■ not equivalence.

Relations — Anti-symmetric

Q38 **A relation R on A is anti-symmetric if:**

(A) aRb and bRa■a=b	**(B)** aRa for all a
(C) aRb■bRa	**(D)** aRb and bRc■aRc

Q38 ANTI-SYMMETRI **✓ (A) aRb and bRa ■ a=b**

SOLUTION:
Anti-symmetric: if both aRb and bRa hold simultaneously, then a must equal b. This is different from 'not symmetric' — the identity relation is both symmetric and anti-symmetric. Example: ≤ on N is anti-symmetric.

Relations — Real-World

Q39 **Relation 'is sister of' among humans is:**

(A) Not reflexive	**(B)** Reflexive
(C) Transitive and reflexive	**(D)** Equivalence

Q39 REAL-WORL **✓ (A) Not reflexive**

SOLUTION:
A person cannot be her own sister ■ **not reflexive**. If A is sister of B, B may or may not be sister of A (depends on gender) ■ may not be symmetric. It can be transitive in some interpretations. Key answer: NOT reflexive.

Relations — Counting

Q40 **If A has 3 elements, number of reflexive relations on A is:**

(A) 2^6	(B) 2^9
(C) 2^3	(D) 3^3

Q40 REFLEXIVE COUN ✓ (A) 2■ = 64

SOLUTION:
A reflexive relation MUST include all n diagonal pairs (a,a). Remaining pairs: n^2–n = 9–3 = 6 pairs can be freely included or not. Number of reflexive relations = 2^6 = **64**. Total relations = 2^9 = 512.

★ PART C — FUNCTIONS (Q 41 – 60)

Functions — Injective

Q41 **A function f:A→B is one-one (injective) if:**

(A) f(a)=f(b)■a=b	(B) Every element of B has a pre-image
(C) f is constant	(D) A=B

Q41 INJECTIVE ✓ (A) f(a)=f(b)■a=b

SOLUTION:
One-one (injective): distinct inputs produce distinct outputs. Formally: f(a)=f(b) ■ a=b (equivalently: a≠b ■ f(a)≠f(b)). Graphically: horizontal line test — any horizontal line cuts the graph at most once.

Functions — Surjective

Q42 **A function f:A→B is onto (surjective) if:**

(A) Range of f = B	(B) f is one-one
(C) A=B	(D) Domain = Co-domain

Q42 SURJECTIVE ✓ (A) Range = B

SOLUTION:
Onto (surjective): every element of the co-domain B has at least one pre-image in A. Equivalently, Range(f) = B. If Range ■ B (proper subset), f is 'into' (not onto).

Functions — Inverse

Q43 **If f(x) = 2x + 3, then $f^{-1}(x)$ = ?**

(A) (x–3)/2	**(B)** (x+3)/2
(C) 2x–3	**(D)** (3–x)/2

Q43 INVERSE FUNCTIO ✓ **(A) (x–3)/2**

SOLUTION:
To find f^{-1}: let y=2x+3. Solve for x: y–3=2x ■ x=(y–3)/2. Replace y with x: **f^{-1}(x)=(x–3)/2**. Verify: $f(f^{-1}(x))$=2·(x–3)/2+3=x–3+3=x ✓.

Functions — Domain

Q44 **The domain of f(x) = √(x–3) is:**

(A) [3,∞)	**(B)** (3,∞)
(C) (–∞,3]	**(D)** All real numbers

Q44 DOMAIN ✓ **(A) [3, ∞)**

SOLUTION:
√(x–3) is defined only when x–3 ≥ 0 (square root of negative is not real). So x ≥ 3. Domain = **[3,∞)**. Note: x=3 is included (√0=0 is valid). Use (for strict inequality, [for ≥.

Functions — Composite

Q45 **If f(x)=x^2 and g(x)=x+1, then (fog)(x) is:**

(A) $(x+1)^2$	**(B)** x^2+1
(C) $x^2(x+1)$	**(D)** 2x+1

Q45 COMPOSIT ✓ **(A) (x+1)²**

SOLUTION:
(fog)(x) = f(g(x)) = f(x+1) = $(x+1)^2$. Note: fog means 'f after g' — first apply g, then f. Compare: (gof)(x)=g(f(x))=g(x^2)=x^2+1 (different!). Order matters in composition.

Functions — Range

Q46 **The range of $f(x) = \sin x$ is:**

(A) $[-1,1]$	(B) $(-1,1)$
(C) $[0,1]$	(D) $(0,1)$

Q46 TRIG RANGE ✓ **(A) $[-1, 1]$**

SOLUTION:
$\sin x$ achieves the values -1 (at $x=-\pi/2+2k\pi$) and $+1$ (at $x=\pi/2+2k\pi$), and all values in between. Range = **$[-1,1]$** (closed interval, endpoints included). $\cos x$ also has range $[-1,1]$; $\tan x$ has range R.

Functions — Modulus

Q47 **$f(x) = |x|$ is:**

(A) Not differentiable at x=0, but continuous	(B) Differentiable everywhere
(C) Neither cont. nor diff.	(D) Differentiable at x=0

Q47 MODULUS FUNCTI ✓ **(A) Continuous, not diff. at 0**

SOLUTION:
$|x|$ is continuous everywhere (no breaks/jumps). At $x=0$: left derivative = -1, right derivative = $+1$ (they differ) ■ **not differentiable at x=0**. Geometrically: sharp corner at origin. For $x\neq0$, $|x|$ is smooth.

Functions — Bijective

Q48 **If $f:R\to R$, $f(x)=x^3$, then f is:**

(A) Bijective	(B) Only injective
(C) Only surjective	(D) Neither

Q48 BIJECTIVE ✓ **(A) Bijective**

SOLUTION:
$f(x)=x^3$ is strictly increasing on R (one-one ✓). For every $y\in R$, $x=y^{1/3}\in R$ exists (onto ✓). Both conditions satisfied ■ **bijective**. Note: $f(x)=x^2$ is neither on R (not one-one, not onto for negatives).

Functions — Domain

Q49 **The domain of f(x) = log(x–1) is:**

(A) $(1,\infty)$	(B) $[1,\infty)$
(C) $(0,\infty)$	(D) R

Q49 LOG DOMAI ✓ **(A) $(1, \infty)$**

SOLUTION:
log is defined only for **strictly positive** arguments. $x-1 > 0$ ■ $x > 1$. Domain = **$(1,\infty)$** (open interval, x=1 excluded since log(0) is undefined). Compare: $\sqrt{(x-1)}$ requires $x \geq 1$, giving $[1,\infty)$.

Functions — GIF

Q50 **f(x) = [x] (greatest integer function). Then f(2.9) = ?**

(A) 2	(B) 3
(C) 2.9	(D) –3

Q50 FLOOR FUNCTIC ✓ **(A) 2**

SOLUTION:
[x] = greatest integer **less than or equal to** x. [2.9] = 2 (since $2 \leq 2.9 < 3$). [–2.3]=–3 (since $-3 \leq -2.3 < -2$). [3]=3. This is the **floor function**, denoted ■x■. Widely tested in NDA.

Functions — Self-Inverse

Q51 **If f(x)=(x+1)/(x–1), then f(f(x)) = ?**

(A) x	(B) 1/x
(C) x+1	(D) x–1

Q51 SELF-INVERSE ✓ **(A) x**

SOLUTION:
f(f(x)) = f((x+1)/(x–1)) = [((x+1)/(x–1))+1] / [((x+1)/(x–1))–1] = [(x+1+x–1)/(x–1)] / [(x+1–x+1)/(x–1)] = (2x)/(2) = **x**. Such functions (f■f=identity) are called **involutions**.

Functions — Bijections

Q52 **The number of bijections from A={1,2,3} to B={a,b,c} is:**

(A) 6	(B) 3
(C) 9	(D) 8

Q52 COUNTING BIJECTION ✓ (A) 6

SOLUTION:
A bijection is a one-one onto mapping. Number of bijections from A to B (with |A|=|B|=n) = n! = 3! = 3×2×1 = **6**. These are the permutations of B.

Functions — Constant

Q53 **f:R→R defined by f(x)=3 (constant) is:**

(A) Many-one and into	(B) Bijective
(C) One-one	(D) Onto

Q53 CONSTANT FUNCTI ✓ (A) Many-one and into

SOLUTION:
Every input maps to 3 ■ **many-one** (not injective). Range={3}≠R ■ **into** (not surjective). Constant functions are neither injective nor surjective (unless domain and codomain are both single-element sets).

Functions — Quadratic

Q54 **If $f(x)=x^2-4x+4$, the minimum value of f is:**

(A) 0	(B) 4
(C) –4	(D) 2

Q54 MINIMUM VALU ✓ (A) 0

SOLUTION:
$f(x) = x^2-4x+4 = (x-2)^2$. Since $(x-2)^2 \geq 0$ for all real x, the minimum value is **0**, achieved at x=2. This is the vertex of the upward-opening parabola.

Functions — Inverse

Q55 **The inverse of $f(x)=e^x$ is:**

(A) $\log_e x$	(B) e^{-x}
(C) $1/e^x$	(D) x

Q55 EXP-LOG INVERS ✓ **(A) ln x**

SOLUTION:
$f(x)=e^x$ and $f^{-1}(x)=\ln x$ (natural logarithm). Verify: $f(f^{-1}(x))=e^{\ln x}=x$ ✓ and $f^{-1}(f(x))=\ln(e^x)=x$ ✓. Domain of ln x = $(0,\infty)$; domain of e^x = R.

Functions — Inverse Value

Q56 **$f(x)=(2x-1)/(x+3)$. Find $f^{-1}(1)$:**

(A) 4	(B) –4
(C) 1	(D) 3

Q56 INVERSE EVALUATIC ✓ **(A) 4**

SOLUTION:
$f^{-1}(1)$ = value of x where $f(x)=1$. Set $(2x-1)/(x+3)=1$ ■ $2x-1=x+3$ ■ x=**4**. Verify: $f(4)=(8-1)/(4+3)=7/7=1$ ✓.

Functions — Even/Odd

Q57 **A function f is even if:**

(A) $f(-x)=f(x)$	(B) $f(-x)=-f(x)$
(C) $f(0)=0$	(D) f is one-one

Q57 EVEN FUNCTIO ✓ **(A) f(–x) = f(x)**

SOLUTION:
Even function: $f(-x)=f(x)$ — symmetric about y-axis. Examples: x^2, cos x, |x|. **Odd function:** $f(-x)=-f(x)$ — symmetric about origin. Examples: x^3, sin x, x. Note: $f(0)=0$ is necessary for odd functions but not sufficient.

Functions — Periodic

Q58 **Period of f(x)=sin(2x) is:**

(A) π	(B) 2π
(C) $\pi/2$	(D) 4π

Q58 PERIOD ✓ (A) π

SOLUTION:
Period of sin(kx) = $2\pi/k$. For k=2: period = $2\pi/2 = \pi$. The function completes one full cycle in π radians. Compare: sin x has period 2π; sin(x/2) has period 4π.

Functions — Exponential

Q59 **If $f(x)=2^x$, then f(x+y) = ?**

(A) f(x)·f(y)	(B) f(x)+f(y)
(C) f(xy)	(D) f(x–y)

Q59 EXPONENTIAL LA ✓ (A) f(x)·f(y)

SOLUTION:
$f(x+y)=2^{x+y}=2^x\cdot 2^y=f(x)\cdot f(y)$. This is the **law of exponents**: $a^{m+n}=a^m\cdot a^n$. Exponential functions convert addition to multiplication — the basis of logarithms.

Functions — Composite

Q60 **The composite (gof)(x) where f(x)=x+2 and $g(x)=x^2$ is:**

(A) $(x+2)^2$	(B) x^2+2
(C) $x^2(x+2)$	(D) x+4

Q60 COMPOSITE go ✓ (A) $(x+2)^2$

SOLUTION:
$(gof)(x)=g(f(x))=g(x+2)=(x+2)^2$. **gof means apply f first, then g.** Compare: $(fog)(x)=f(g(x))=f(x^2)=x^2+2$. The two composites are different!

★ QUICK REFERENCE — KEY FORMULAE & FACTS

SETS

- $n(A\cup B) = n(A)+n(B)-n(A\cap B)$
- $n(A\cup B\cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(A\cap C)+n(A\cap B\cap C)$
- De Morgan's: $(A\cup B)'=A'\cap B'$, $(A\cap B)'=A'\cup B'$
- $A-B = A\cap B'$, A■B $= (A-B)\cup(B-A) = (A\cup B)-(A\cap B)$
- $n(A\times B) = n(A)\cdot n(B)$, $n(P(A)) = 2^{n(A)}$

RELATIONS

- Reflexive: $(a,a)\in R$ for all $a\in A$
- Symmetric: $(a,b)\in R$ ■ $(b,a)\in R$
- Transitive: $(a,b)\in R$ and $(b,c)\in R$ ■ $(a,c)\in R$
- Equivalence = Reflexive + Symmetric + Transitive
- Bell numbers $B(1)=1$, $B(2)=2$, $B(3)=5$, $B(4)=15$
- Reflexive relations on n-set: $2^{(n^2-n)}$
- Total relations on n-set: $2^{(n^2)}$

FUNCTIONS

- Injective (one-one): $f(a)=f(b)$ ■ $a=b$
- Surjective (onto): Range(f) = Codomain
- Bijective = Injective + Surjective
- Number of bijections $A\to B$ ($|A|=|B|=n$) $= n!$
- $fog(x) = f(g(x))$ [apply g first, then f]
- Domain of $\log(x-a)$: $x > a$ | Domain of $\sqrt{(x-a)}$: $x \geq a$
- Even: $f(-x)=f(x)$ | Odd: $f(-x)=-f(x)$
- Period of $\sin(kx)$ or $\cos(kx) = 2\pi/k$

★ SPACE FOR ROUGH WORK